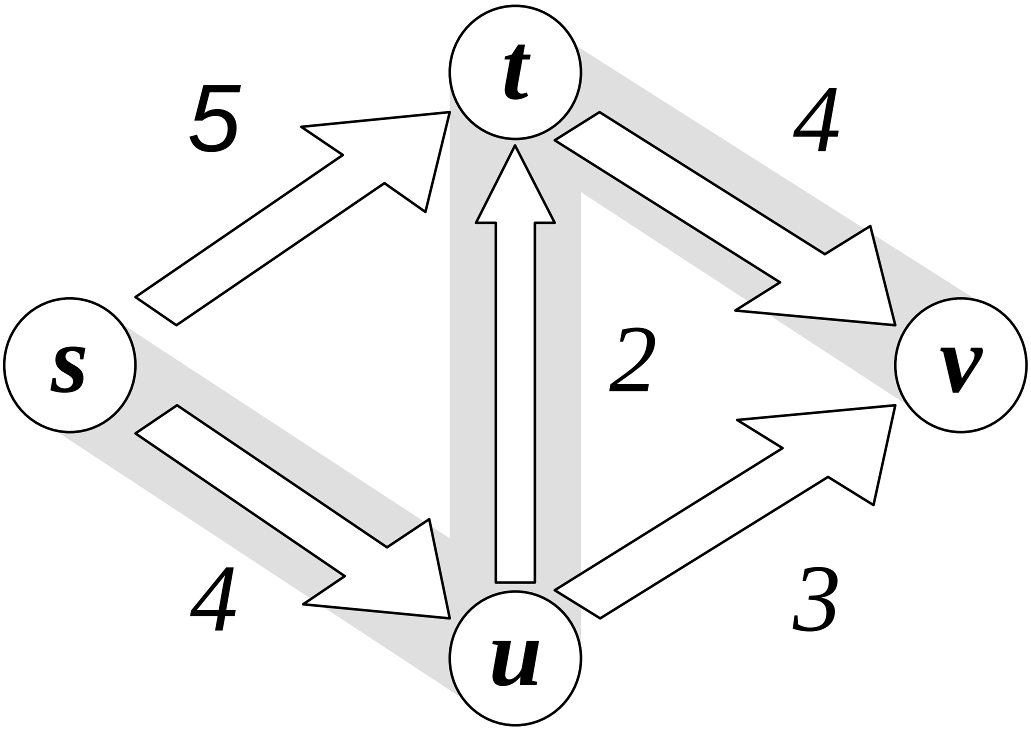
Network Techniques - I.



PERT$^{\mathbf{time/cost}}$ **:**

(**P**rogram **E**valuation & **R**eview **T**echnique)

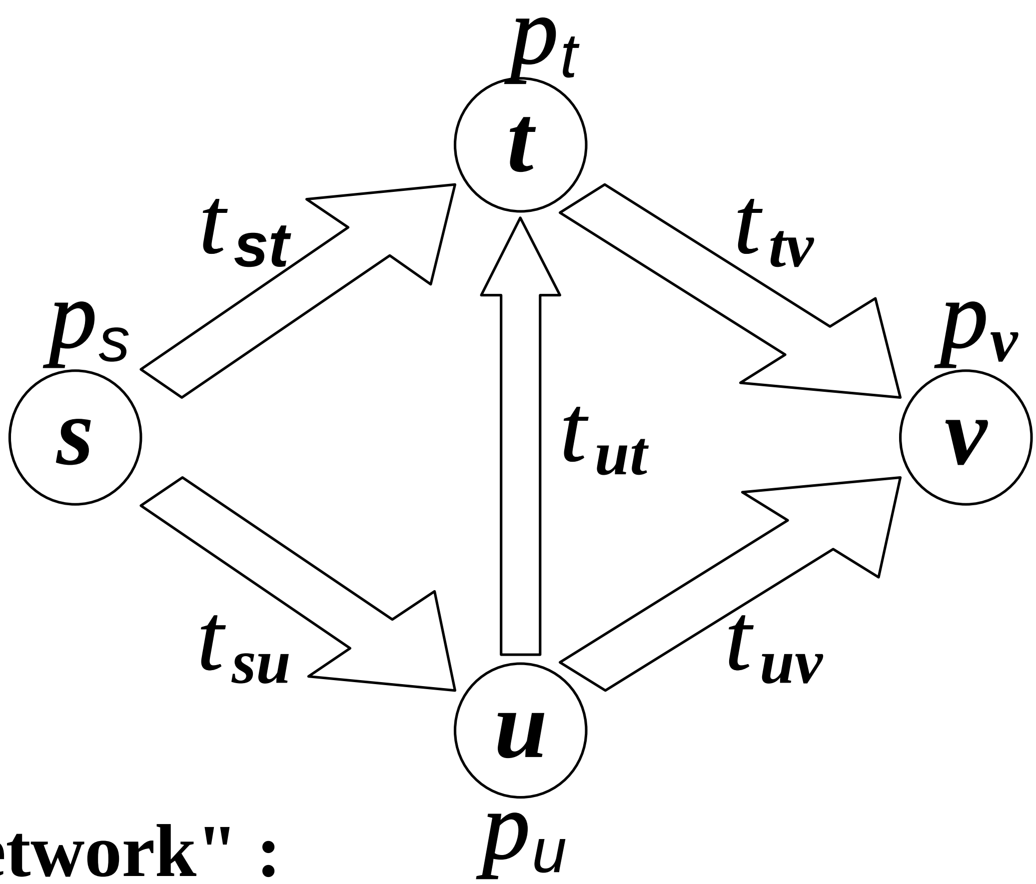
Event-on-node typed project-model with probabilistic (stochastic) data set as weights (inp) and time potentials (outp)

CPM$^{\mathbf{time/cost}}$ **:**

(**C**ritical **P**ath **M**ethod)

Activity-on-arrow typed project-model with discrete (deterministic) data set as weights (inp) and time potentials (outp)

PERT/CPM Graph-restrictions



"Network" :

Connected weighted directed graph with a single source and a single sink but with no loops and no negative weights.

"One-to-one correspondence" :

Each identified particle is presented in the model by its only single representative

"Edges as related pairs of nodes" :

Between two nodes at maximum one single directed edge (arrow) is allowed

Program Evaluation & Review Technique (PERT)

1958 : US Navy, Polaris Program, Farard

Nodes :

events, states, "mile-stones", phases of progression

Edges :

Activities ("sub-projects") with closely not identified (technical) contents (R&D)

Parameters (weights) :

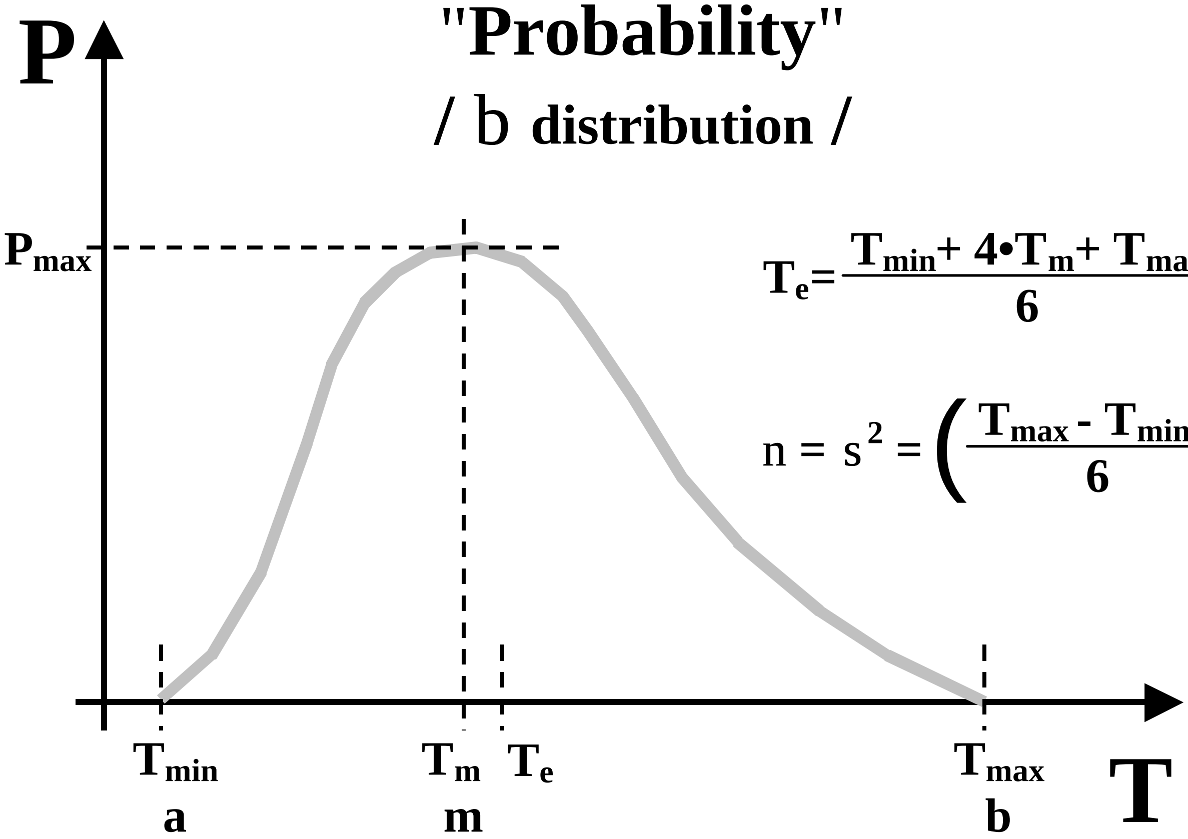
<u>Probabilistic variables</u> ("time-spans") of b **distribution** based on triplex estimates

Aims :

Predict timing of milestones and overall execution time of a project, together with indices of uncertainty ("deviation").
To check feasibility of a Schedule.

"Probability"

/ b distribution /

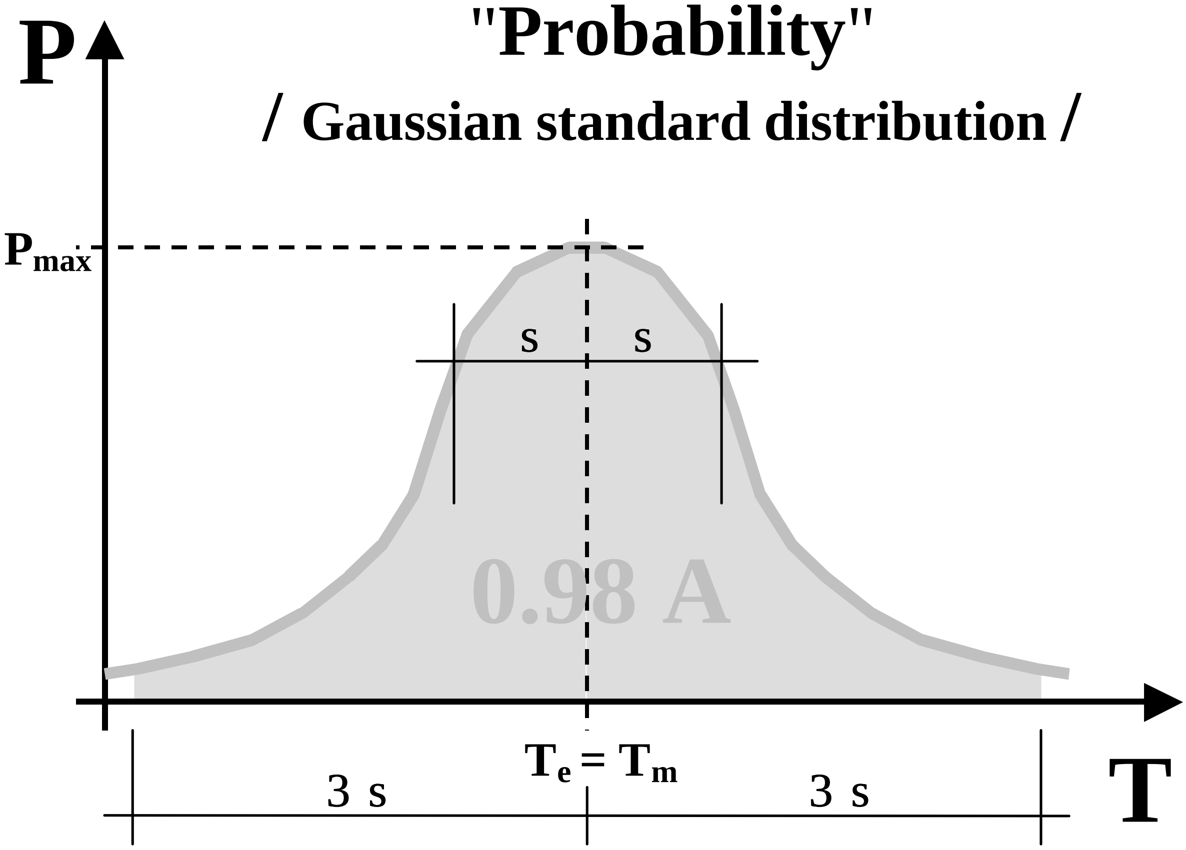


$$T_e = \frac{T_{min} + 4 \bullet T_m + T_{max}}{6}$$

$$n = s^2 = \left(\frac{T_{max} - T_{min}}{6}\right)^2$$

"Probability"

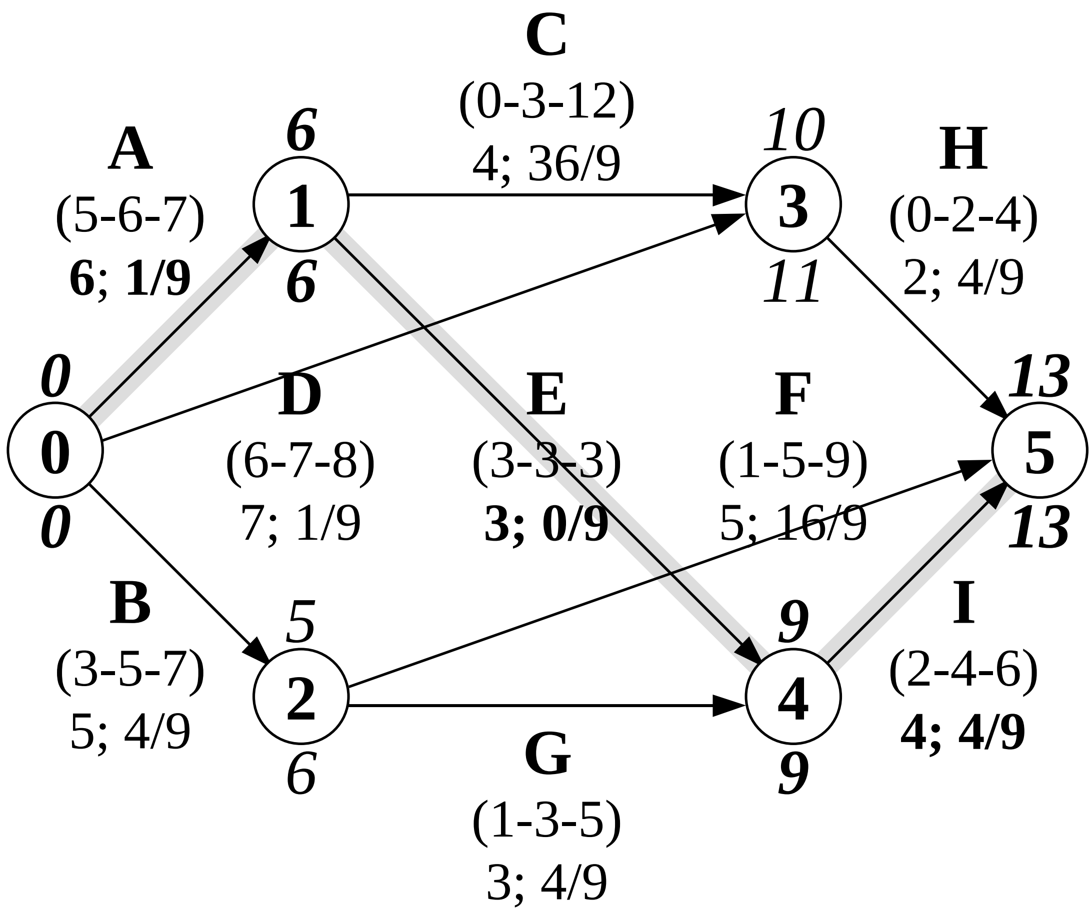
/ Gaussian standard distribution /



(PERT) Problem:

What is the probability of matching the 12 tu Schedule of the project below ?

ID
(a-m-b)
m_e; n



$$\mu_e = \frac{a + 4 \bullet m + b}{6}$$

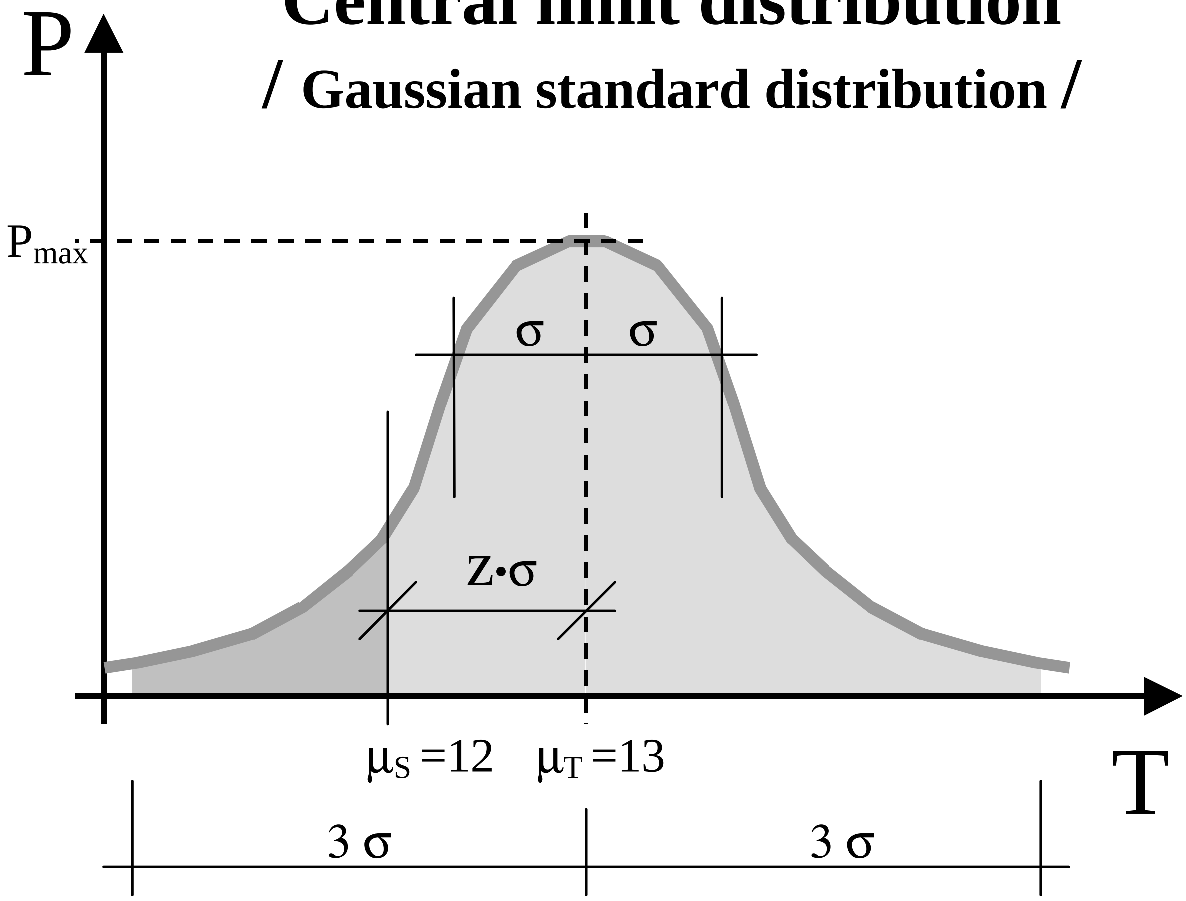
$$\nu = \sigma^2 = \left(\frac{b - a}{6}\right)^2$$

$$\mu_T = 13$$

$$\nu_T = 5/9$$

Central limit distribution

/ Gaussian standard distribution /



$$Z = \frac{\mu_S - \mu_T}{\sqrt{\nu_T}} = \frac{12-13}{\sqrt{5/9}} = -1.3416$$

Z	CP	Z	CP
- 2.0	0.02	+ 0.1	0.54
- 1.5	0.07	+ 0.2	0.58
- 1.3	0.10	+ 0.3	0.62
- 1.0	0.16	+ 0.4	0.66
- 0.9	0.18	+ 0.5	0.69
- 0.8	0.21	+ 0.6	0.73

CP ≈ 9 %

Critical Path Method (CPM^{time})

1957 : USA, E. I. du Pont de Nemours,
James E. Kelly, Morgan R. Walker

Nodes :

links, relations, direct precedences

Edges :

Activities ("sub-projects") with well identified (technical) content, direct precedences (see:"dummy activities")

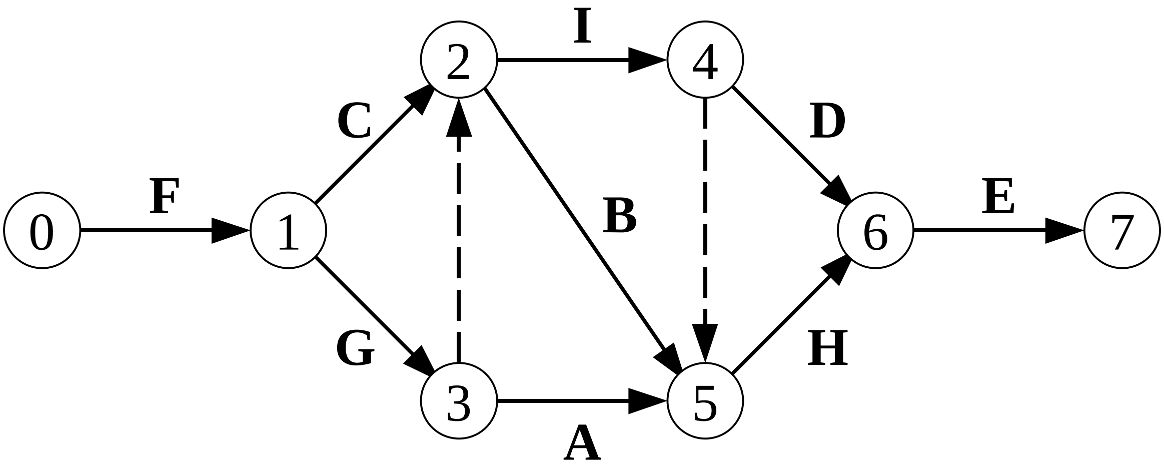
Parameters (weights) :

Activity durations, elapsed times and deadlines (deterministic variables)

Aims :

Identify project elements significant ("dominant"/"critical") in timing of a project. Determine mile-stones and deadlines for execution.
Indicate degree of freedom ("float") of a schedule for parts- and total of a project.

CPM / PERT graph-structure - operative informations -



List of direct precedences

A,B,I < H

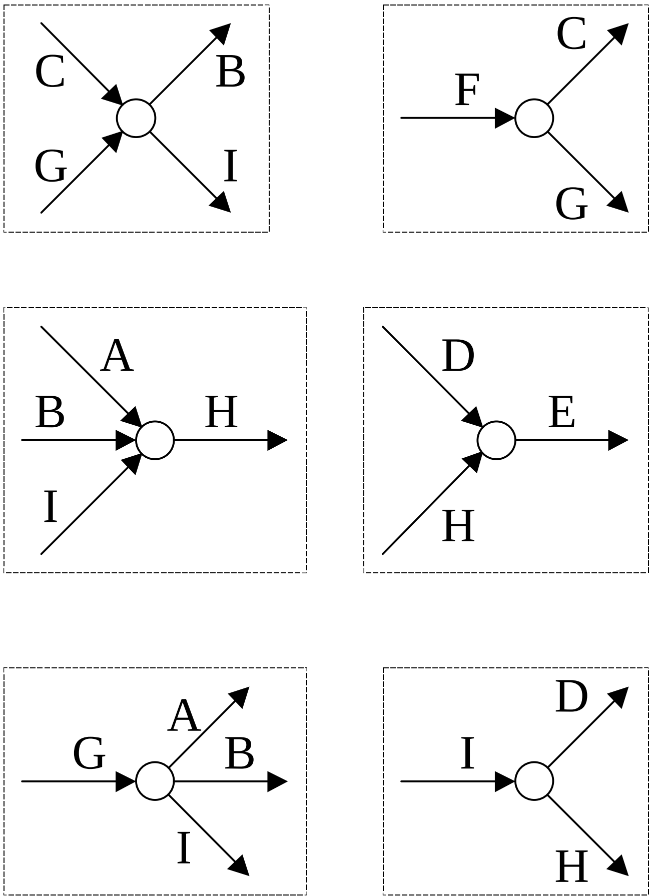
C,G < B,I

D,H < E

F < C,G

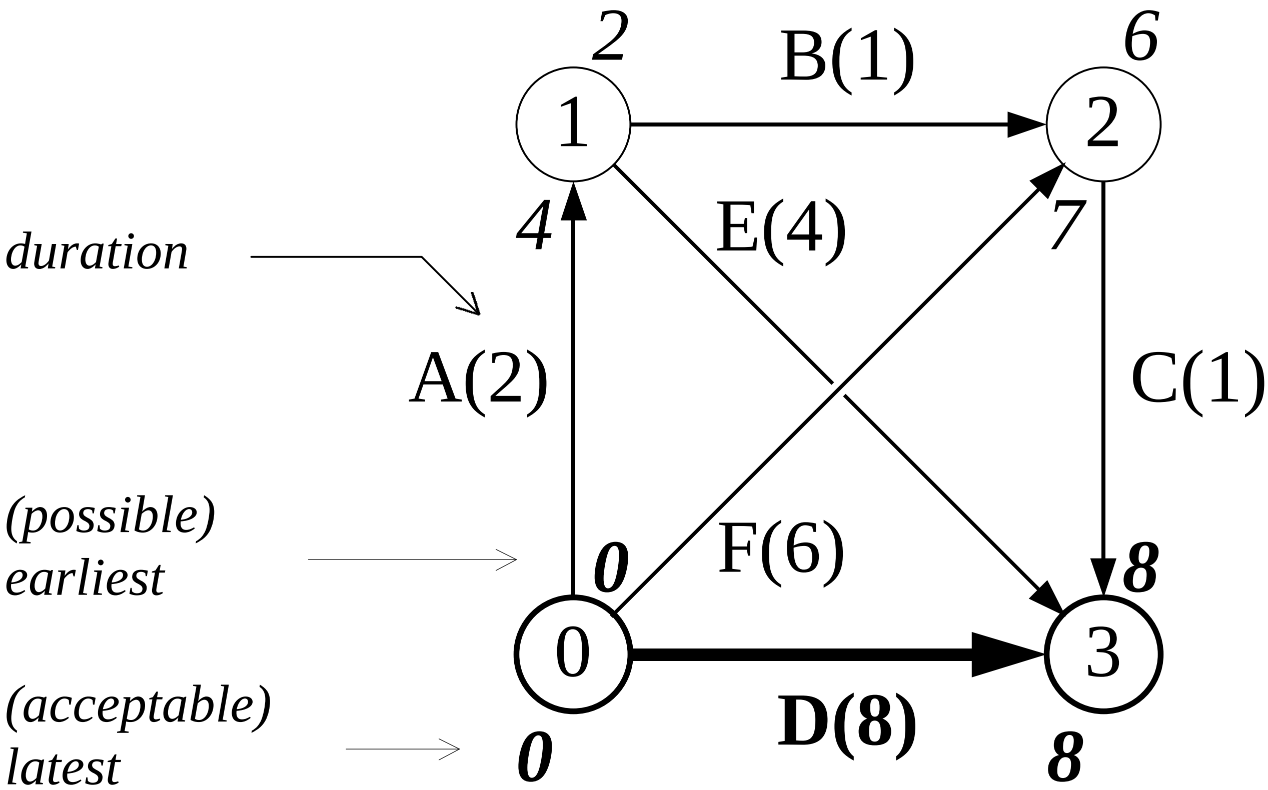
G < A,B,I

I < D,H



Operative informations

(CPM^{time}) Problem:



ID	D	ES	EF	LS	LF	TF	FF	CF	IF
A	**2**	*0*	*2*	*2*	*4*	2	0	2	0
B	**1**	*2*	*3*	*6*	*7*	4	3	2	1

"Critical Path" :

Sub-graph of a graph constituted by nodes – and dominant edges between – at which the earliest schedule equals to the latest one. ("… project elements with no float …")

Sub-graph constituted by the longest paths between the only source and the only sink.

"<u>T</u>otal <u>F</u>loat" (of an activity) :

Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the *<u>early finish of the project</u>* assuming that all its (dominant) predecessors can be performed by their *<u>early</u>* schedules. ("… no delay before, maximum delay after …")

"<u>F</u>ree <u>F</u>loat" (of an activity) :

Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the *<u>early schedule of any activity</u>* assuming that all its (dominant) predecessors can be performed by their *<u>early</u>* schedules. ("… no delay before, no delay after …")

"<u>C</u>onditional <u>F</u>loat" (of an activity) :

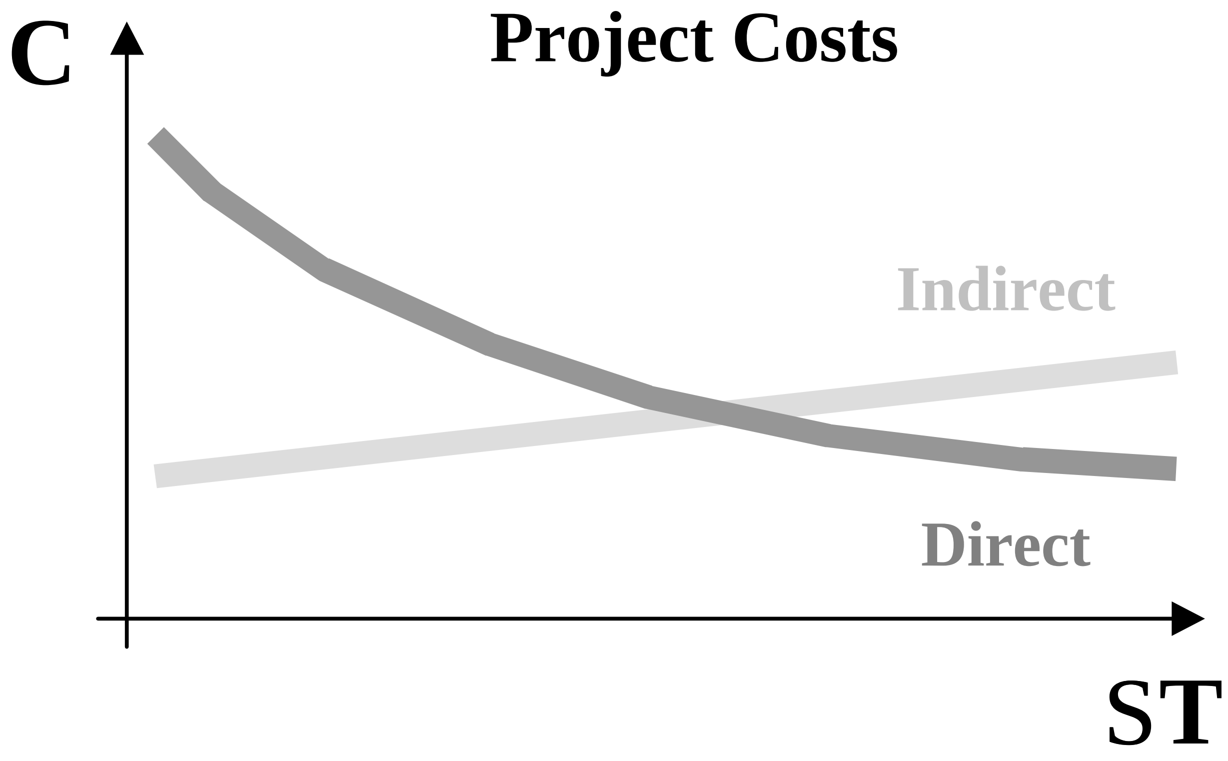
Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the *<u>early finish of the project</u>* assuming that all its (dominant) predecessors can be performed by their *<u>late</u>* schedules. ("… maximum delay before, maximum delay after …")

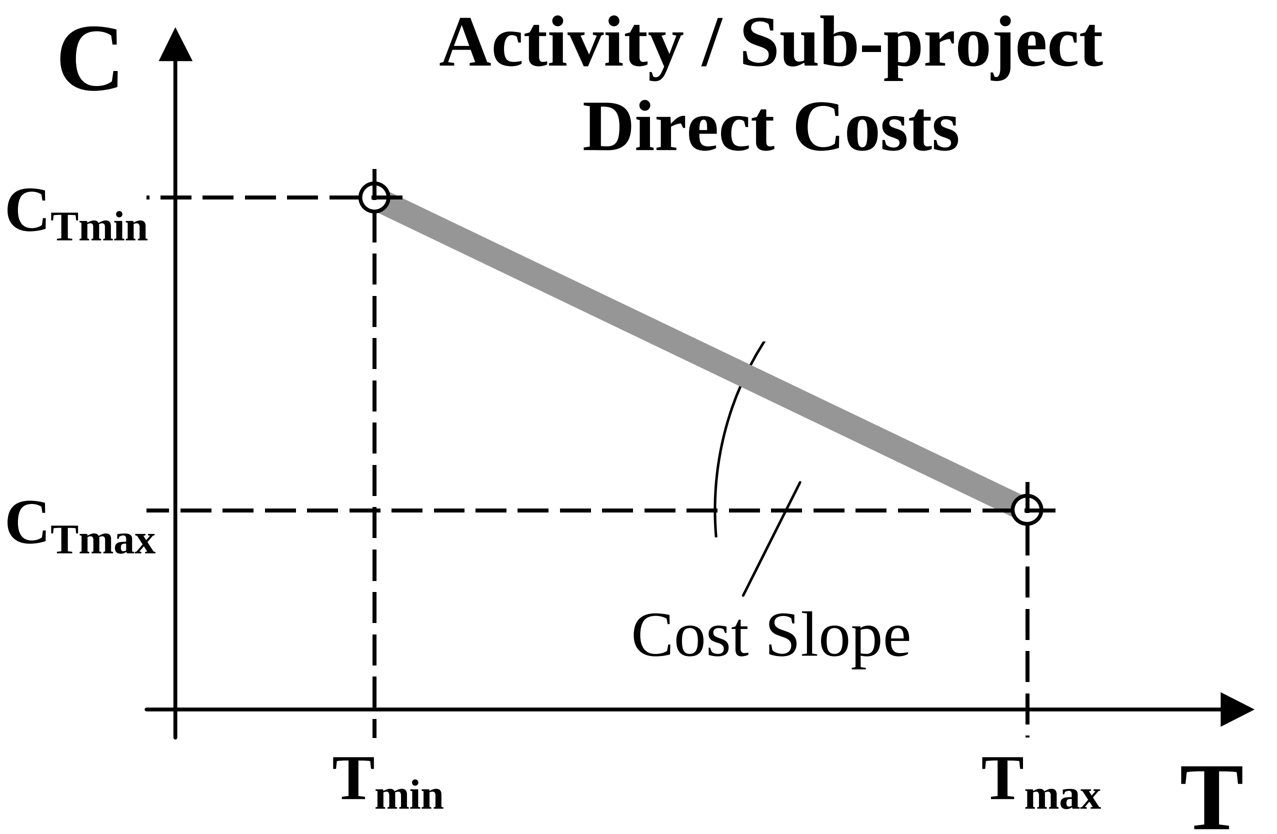
"<u>I</u>ndependent <u>F</u>loat" (of an activity) :

Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the *<u>early schedule of any activity</u>* assuming that all its (dominant) predecessors can be performed by their *<u>late</u>* schedules.
("… maximum delay before, no delay after …")

(Non-negative values interpreted only !)

CPM^{cost} *(CPM cost model)*

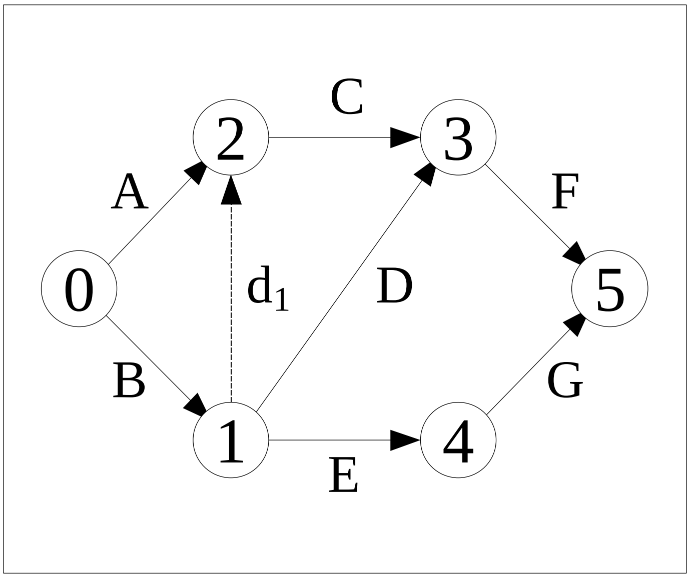


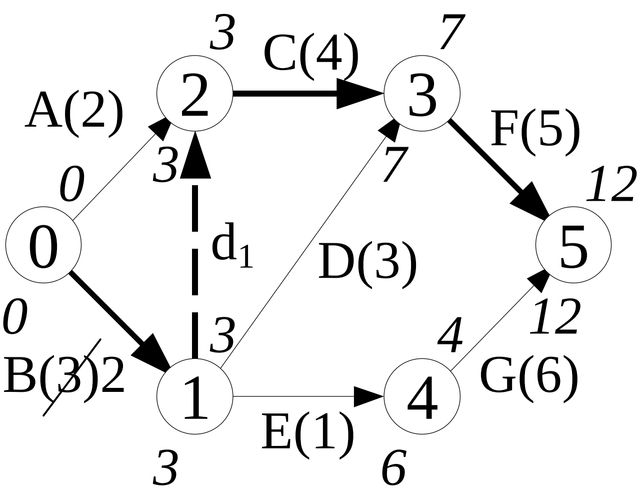


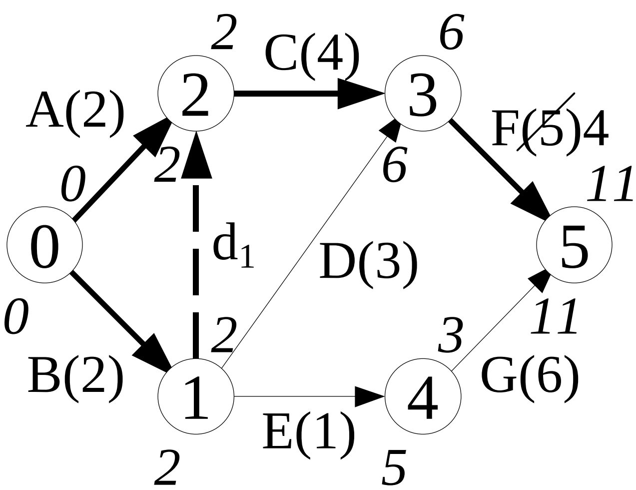
(CPM^cost) Problem :

What is the minimum of "direct" cost of the project below associating an overall execution time not longer than 10 tu ?

ID	Normal		Crash		
	time	cost	time	cost	CS
A	2	120	1	200	80
B	3	80	1	200	60
C	4	100	2	350	125
D	3	150	3	150	-
E	1	250	1	250	-
F	5	130	2	460	110
G	6	80	5	110	30



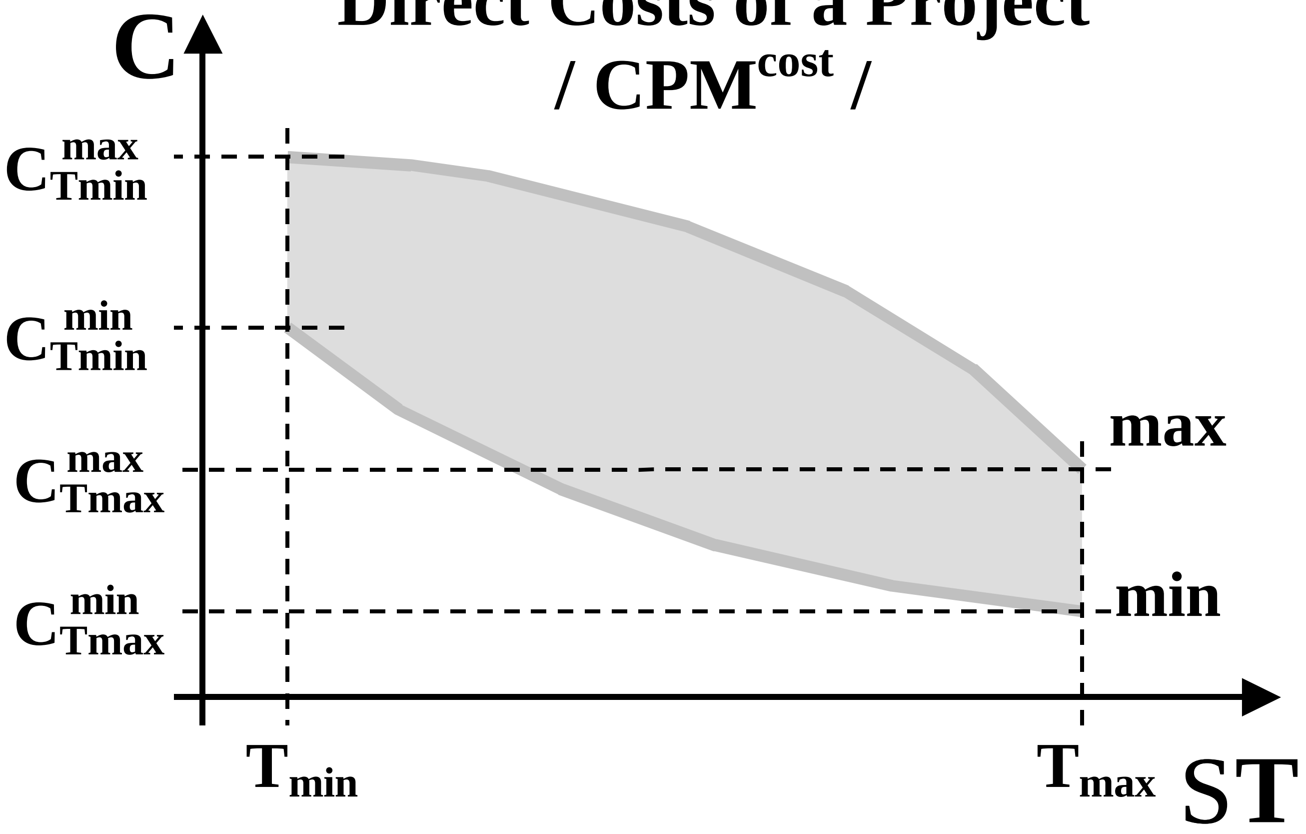




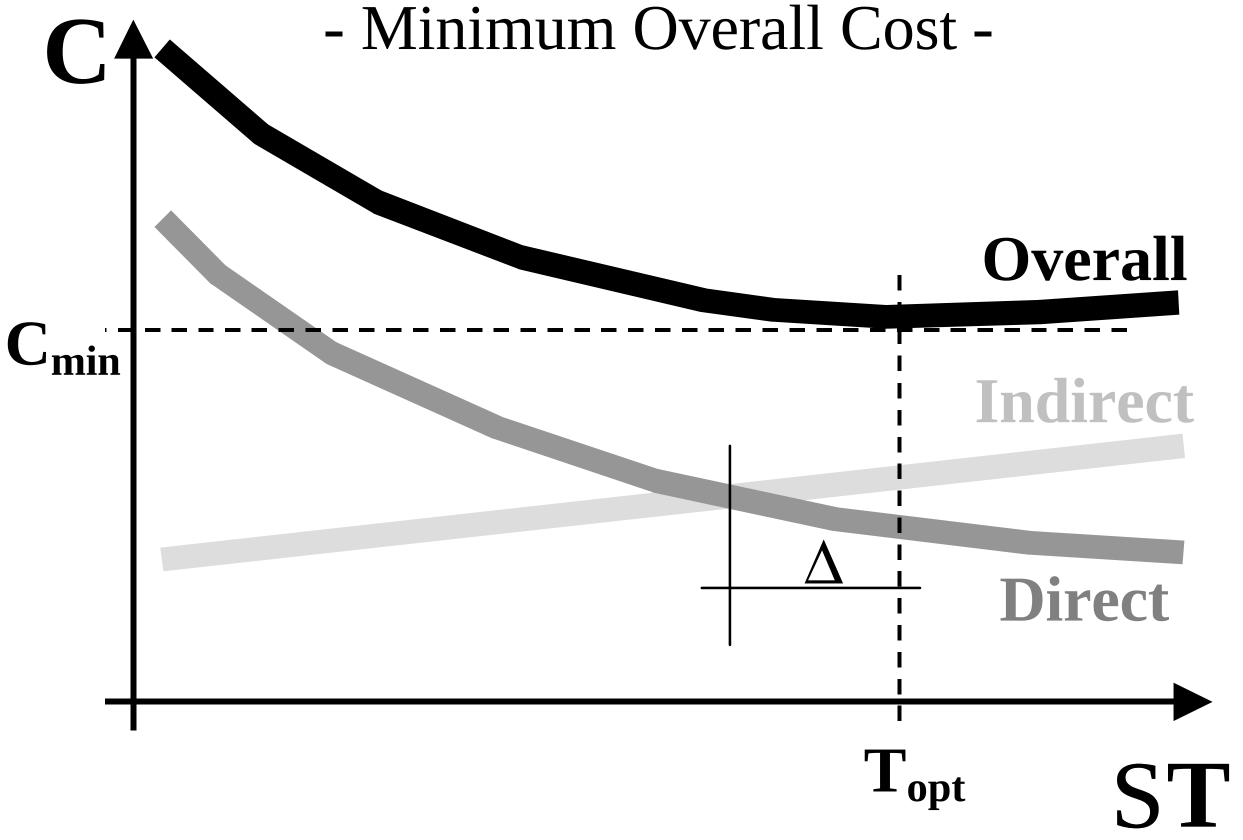
$$C_{11} = C_{12} + CS_B = 910 + 60 = 970$$

$$C_{10} = C_{11} + CS_F = 970 + 110 = 1080$$

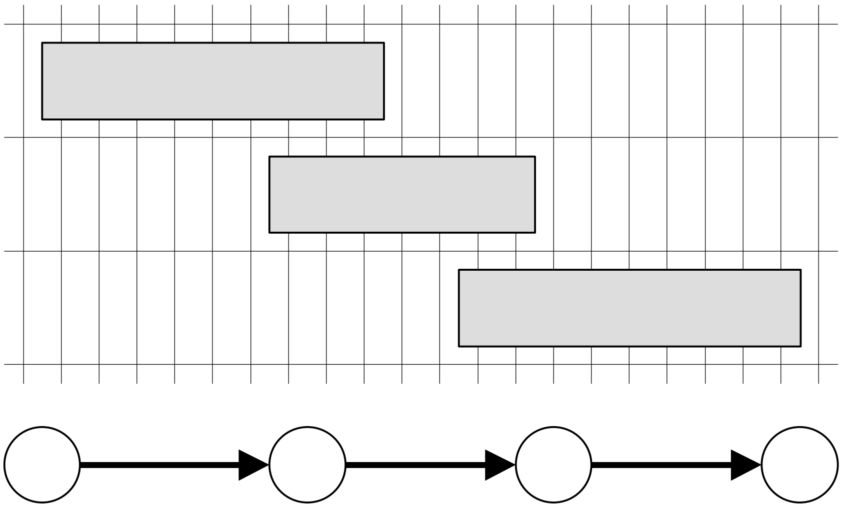
Direct Costs of a Project
/ CPM^{cost} /



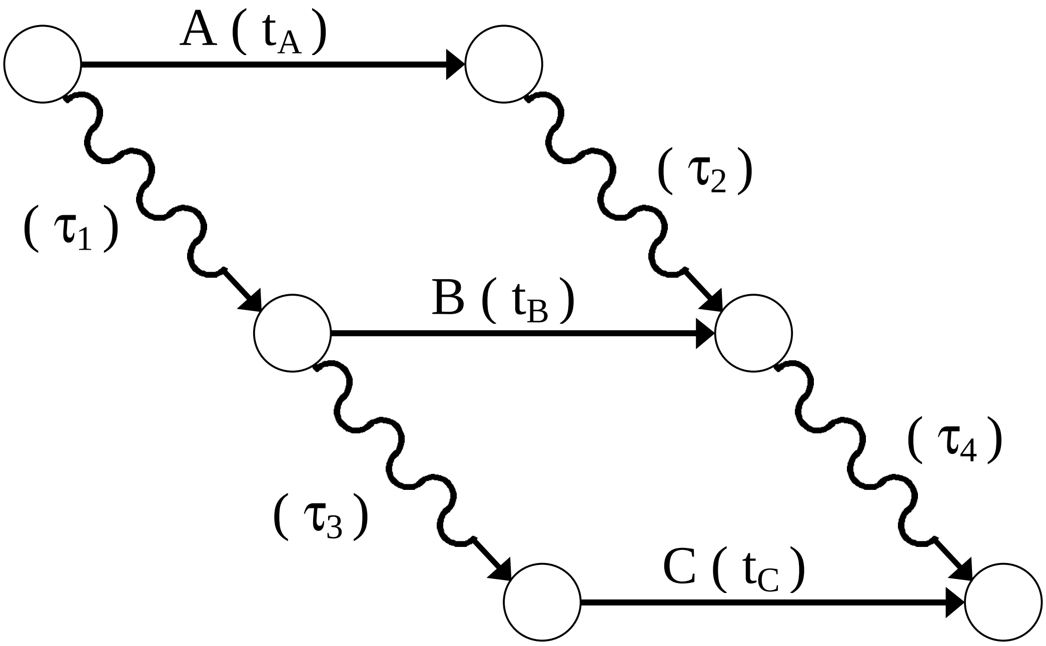
Optimal Project Duration
- Minimum Overall Cost -



CPM^{time+}



Problem: CPM – difficulties with overlapping
Solution: Parameters on "dummy activities"

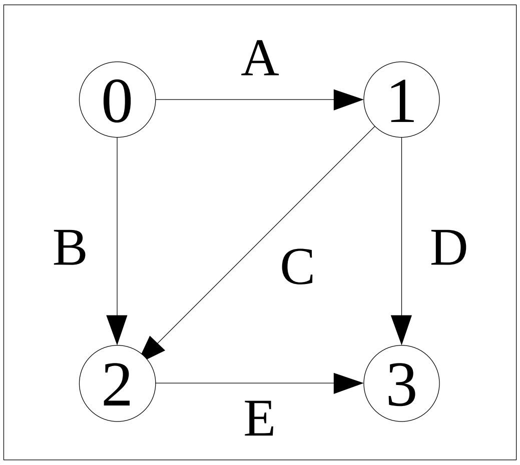


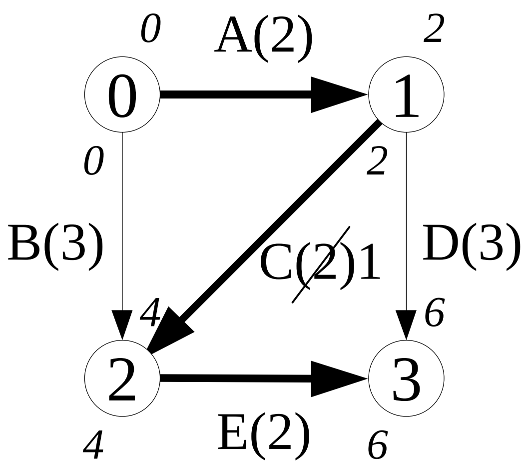
Negative parameters still forbidden. Further problems with open networks and fixed durations

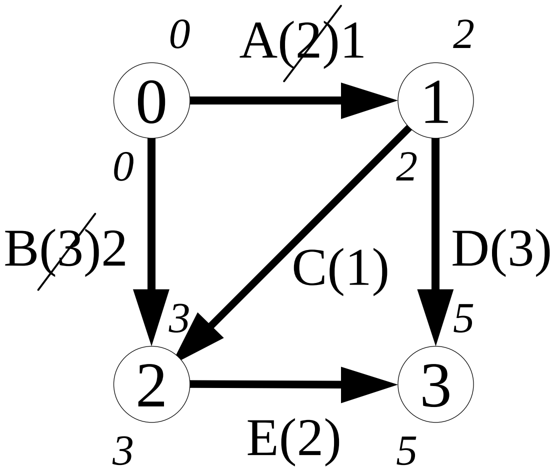
$(\text{CPM}^{\text{cost}})$ Problem* :

What is the minimum of "direct" cost of the project below associating an overall execution time not longer than 4 tu ?

ID	Normal		Crash		
	time	cost	time	cost	CS
A	2	1	1	3	2
B	3	1	1	5	2
C	2	1	1	2	1
D	3	1	1	5	2
E	2	1	1	3	2







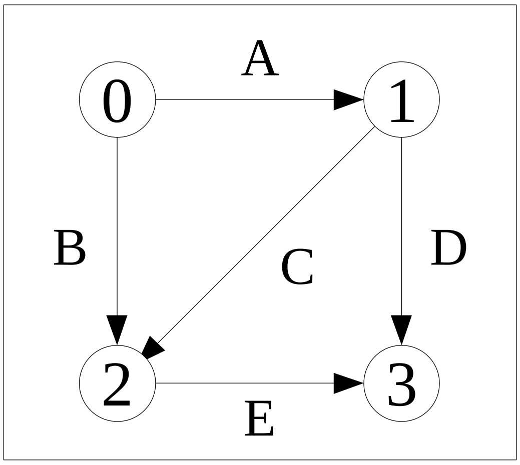
$C_5 = C_6 + CS_C = 5 + 1 = 6$

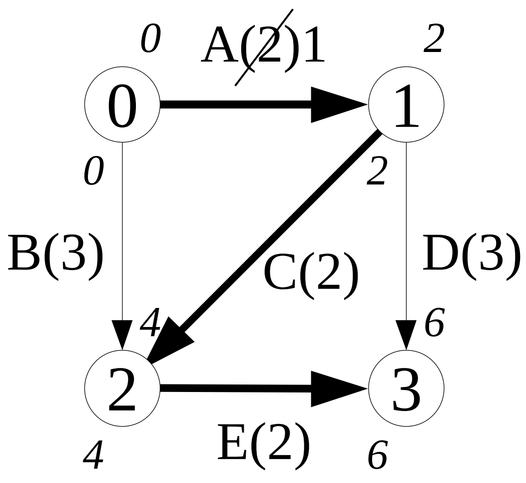
$C_4 = C_5 + CS_{A+B} = 6 + 4 = 10$?

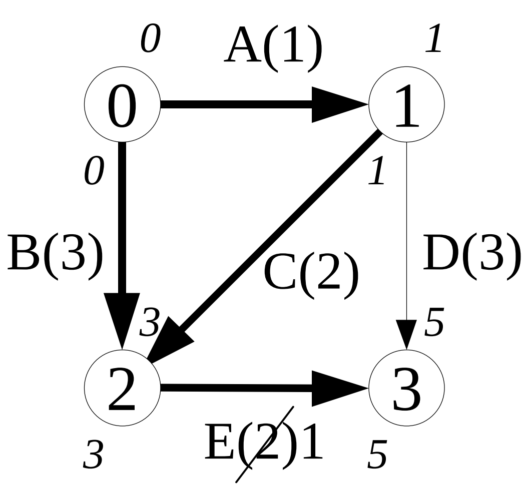
(CPM^{cost}) Problem* :

What is the minimum of "direct" cost of the project below associating an overall execution time not longer than 4 tu ?

ID	Normal		Crash		CS
	time	cost	time	cost	
A	2	1	1	3	2
B	3	1	1	5	2
C	2	1	1	2	1
D	3	1	1	5	2
E	2	1	1	3	2







$C_5 = C_6 + CS_A = 5 + 2 = 7$ (> 6)

$C_4 = C_5 + CS_E = 7 + 2 = 9$ (< 10) !