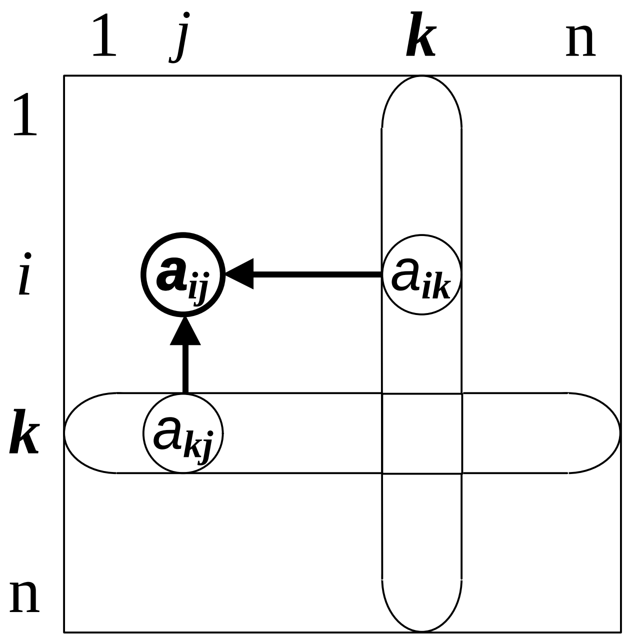
GLOBAL EXAMINATIONS
(all-relation analyses)
OF GRAPHS

Relation: Directed pair of nodes *[i,j]*



Triviality:

If *P[i,k]* path exists on a graph, and *P[k,j]* path also exists, then path *P[i,j]* exists too.

In this context we refer to node ***k*** as a transfer node in relation of *[i,j]*, while we refer all *P[i,j]* pathes together as *[i,j]* accesses ($\boldsymbol{a_{ij}}$).

The set of $\varphi(\underline{\underline{A}})$ transformations

$$\varphi^{0}(\underline{\underline{A}}) = \underline{\underline{A}}$$

$$\varphi^{k}(\underline{\underline{A}}) = \varphi(\varphi^{k-1}(\underline{\underline{A}})) \quad | \quad k = 1, 2, \ldots n$$

Initiating matrix („table of direct accesses”):

Basis („empty” matrix): $a_{ij} = M$ $\forall\, i,j$ **but!:**

At non-weighted graph: $a_{ij} = 1$ **if [i,j] edge exists** $\forall\, i,j$

At weighted a graph: $a_{ij} = t_{ij}$ **if [i,j] edge exists** $\forall\, i,j$

Matrix transformations:

$$a_{ij}^{0} = a_{ij} \quad \forall\, i,j$$

$$a_{ij}^{k} = \begin{cases} \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) & | \quad a_{ik}^{k-1} \neq M;\ a_{kj}^{k-1} \neq M;\ i \neq k;\ j \neq k \\ a_{ij}^{k-1} & \text{otherwise} \end{cases} \quad \forall\, i,j$$

$$k = 1, 2, \ldots n$$

Basic Problems:

Connectivity analysis: $M = 0;\quad \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = 1;$ (non-directed edges)

Dominance analysis: $M = 0;\quad \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = 1;$ (directed edges)

Loop discovering: $M = 0;\quad \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = \max\{a_{ij}^{k-1},\ 2 - a_{ij}^{k-1}\}$

Path-variants’ counting: $M = 0;\quad \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = a_{ij}^{k-1} + (a_{ik}^{k-1} \cdot a_{kj}^{k-1})$

Gravity-point/Center/Span: $M = +\infty;\quad \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = \min\{a_{ij}^{k-1}, a_{ik}^{k-1} + a_{kj}^{k-1}\}$

The longest span: $M = -\infty;\quad \varphi(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = \max\{a_{ij}^{k-1}, a_{ik}^{k-1} + a_{kj}^{k-1}\}$

Floyd-Warshall

(„All-pairs shortest path”)

Initialization :

```
for i:=1 to n do
    for j:=1 to n do begin
        a[i,j]:= w[i,j]*;
        p[i,j]:=0
    end;
```

Scanning :

```
for k:=1 to n do
    for i:=1 to n do
        for j:=1 to n do
            if a[i,k]+a[k,j]<a[i,j] then begin
                a[i,j]:=a[i,k]+a[k,j];
                p[i,j]:=k
            end;
```

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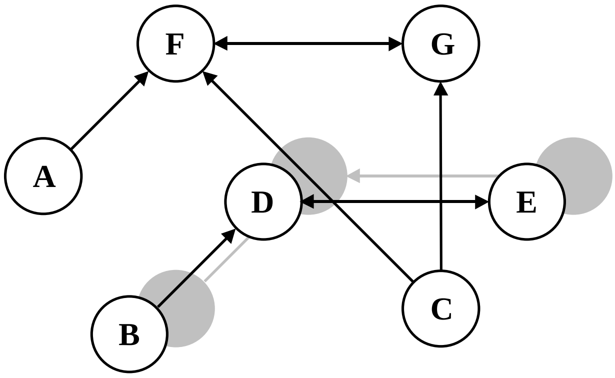
* $w[i,j]:=\tau[i,j]$, if $(i,j)\in A$; $w[i,j]:=M$ otherwise

Warshall, 1959 – finding loops
Floyd, 1962 – all-pairs shortest path

CONNECTIVITY ANALYSIS

$$\left(M = 0;\ \varphi\left(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1} \right) = 1 \right)$$



i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A 1						1	
B 2				1			
C 3						1	1
D 4		1			1		
E 5				1			
F 6	1		1				1
G 7			1			1	

The symmetrical structure matrix

i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A 1						1	
B 2				1			
C 3						1	1
D 4		1			1		
E 5				1			
F 6	1		1			1	1
G 7			1			1	

K = 1

i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A 1						1	
B 2				1			
C 3						1	1
D 4		1		1	1		
E 5				1			
F 6	1		1			1	1
G 7			1			1	

K = 2

i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A 1						1	
B 2		1		1	1		
C 3						1	1
D 4		1		1	1		
E 5		1		1	1		
F 6	1		1			1	1
G 7			1			1	1

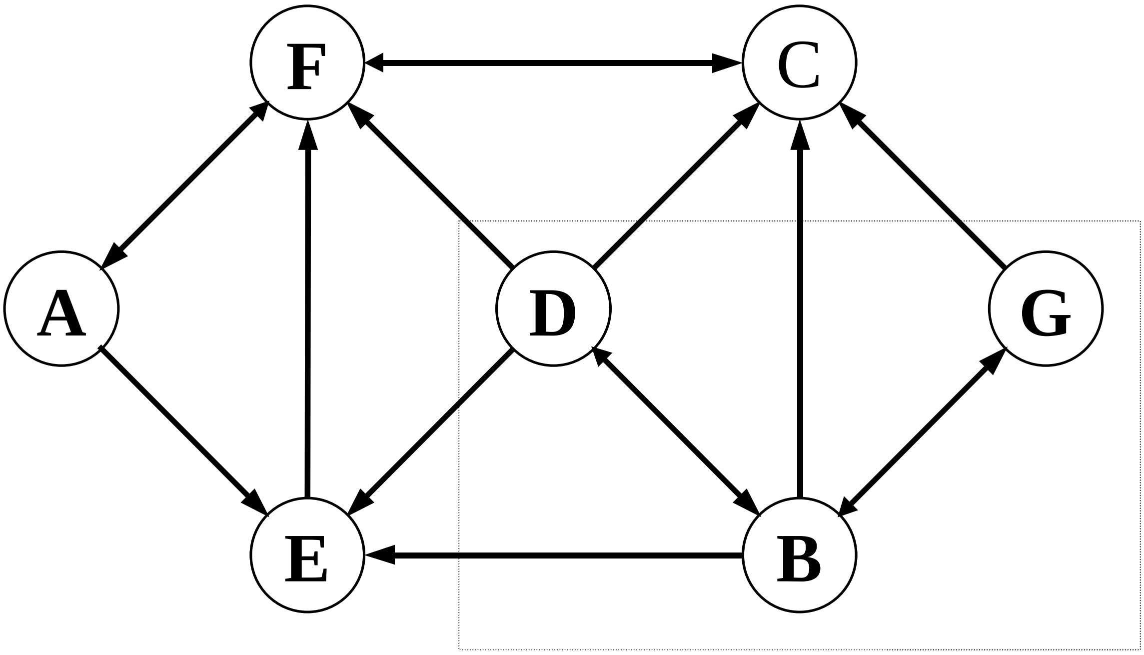
… K = 4 …

i \ j	A 1	C 3	F 6	G 7	B 2	D 4	E 5
A 1	1	1	1	1			
C 3	1	1	1	1			
F 6	1	1	1	1			
G 7	1	1	1	1			
B 2					1	1	1
D 4					1	1	1
E 5					1	1	1

The re-arranged Overall Access Table

DOMINANCE ANALYSIS

$$(\; M = 0; \;\; \varphi \, (a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}) = 1 \;)$$



Dominant Node(s): Node(s) *i* of the graph from which all other nodes of the graph can be accessed. (*P[i,j]* exists in all *[i,j]* *j* ¹ *i* relation)

Dominated Node(s): Node(s) *i* of the graph which can be accessed from all other nodes of the graph. (*P[j,i]* exists in all *[j,i]* *j* ¹ *i* relation.)

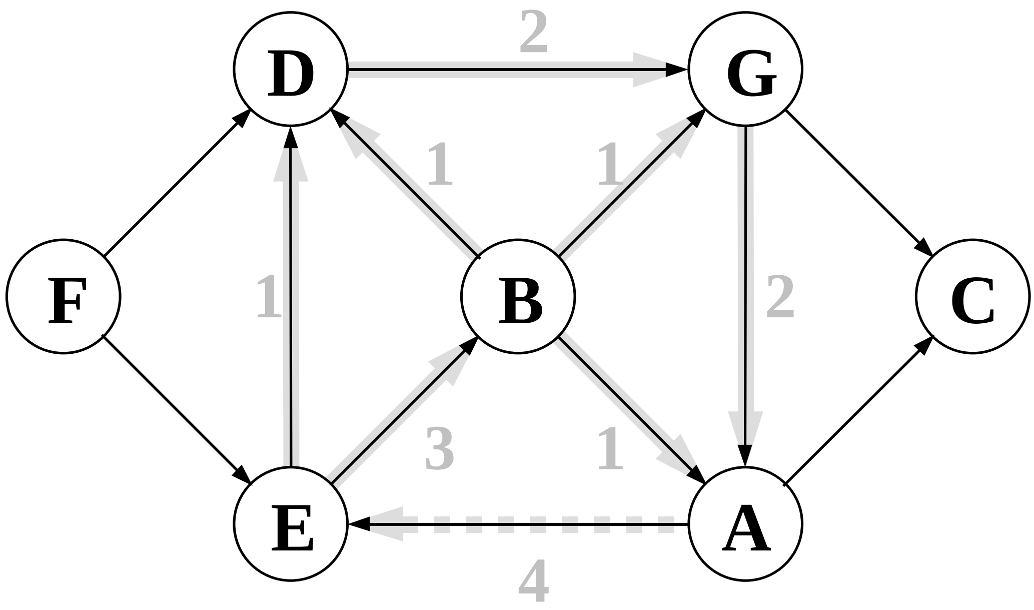
	i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A	1					1	1	
B	2			1	1	1		1
C	3						1	
D	4		1	1		1	1	
E	5						1	
F	6	1		1				
G	7		1	1				

	i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7	
A	1	1		1		1	1		
B	2	1	1	1	1	1	1	1	○
C	3	1		1		1	1		
D	4	1	1	1	1	1	1	1	○
E	5	1		1		1	1		
F	6	1		1		1	1		
G	7	1	1	1	1	1	1	1	○
		○		○		○	○		

Direct- and Overall Access Table with the dominant- and dominated nodes indicated.

LOOP DISCOVERING

$$\left(M = 0;\ \varphi\left(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1} \right) = \max \left\{ a_{ij}^{k-1}, 2 - a_{ij}^{k-1} \right\} \right)$$

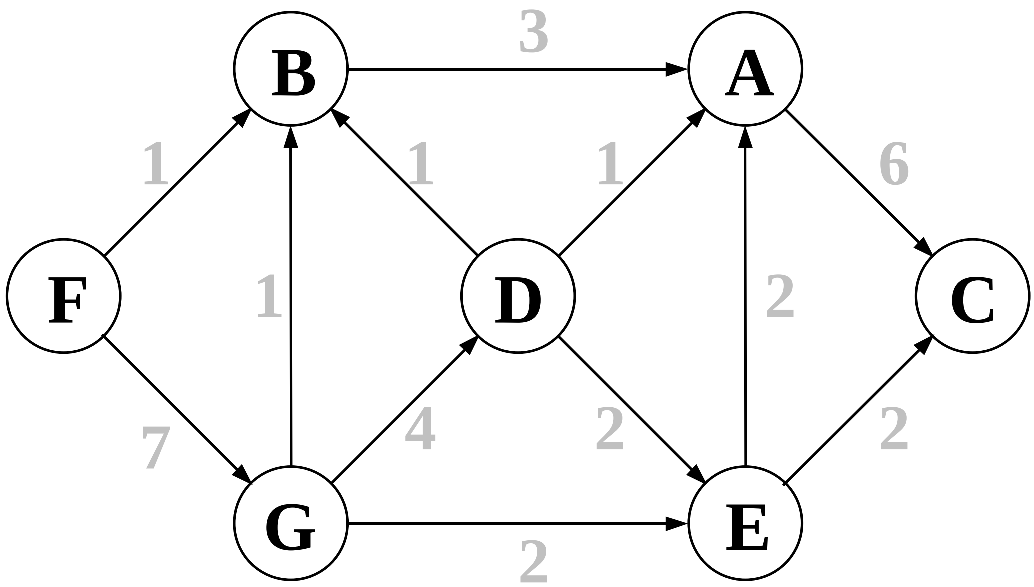


	i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A	1	2	2	1	2	1^{4}		2
B	2	1^{1}	2	2	1^{1}	2		1^{1}
C	3							
D	4	2	2	2	2	2		1^{2}
E	5	2	1^{3}	2	1^{1}	2		2
F	6	2	2	2	1	1		2
G	7	1^{2}	2	1	2	2		2

Overall Access Table with estimated numbers of loops constituted

PATH VARIANTS' COUNTING

$$\left(M = 0;\ \varphi\left(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}\right) = a_{ij}^{k-1} + \left(a_{ik}^{k-1} \cdot a_{kj}^{k-1}\right) \right)$$

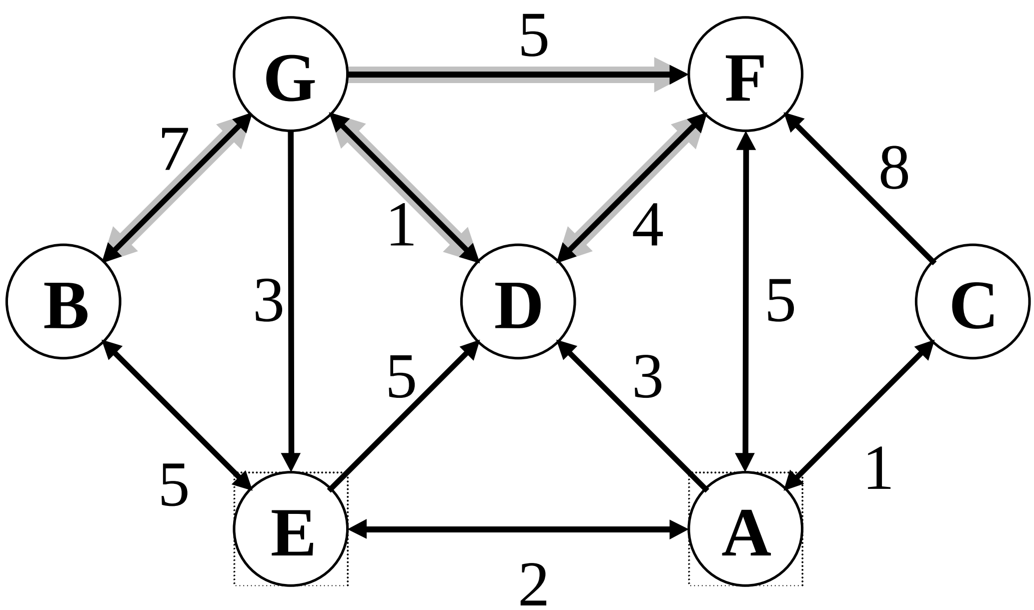


		A	B	C	D	E	F	G
	i \ *j*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
A	*1*			1^{6}				
B	*2*	1^{3}		1				
C	*3*							
D	*4*	3^{1}	1^{1}	4		1^{2}		
E	*5*	1^{2}		2^{2}				
F	*6*	6	3^{1}	**8**	1	2		1^{7}
G	*7*	5	2^{1}	7	1^{4}	2^{2}		

Overall Access Table with number of path-variants constituted in relation *FC*

Gravity-point / Center / Span

$$\left(M = +\infty;\ \varphi\left(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1} \right) = \min\left\{ a_{ij}^{k-1}, a_{ik}^{k-1} + a_{kj}^{k-1} \right\} \right)$$



Gravity-point: Node(s) of the graph from (/to) which the sum of length of shortest pathes to (/from) all other nodes of the graph is the minimum.

Center: Node(s) of the graph from (/to) which the maximum of the length of shortest pathes to (/from) any other nodes of the graph is the minimum.

Span: The Longest of „Shortest Paths” throughout the graph

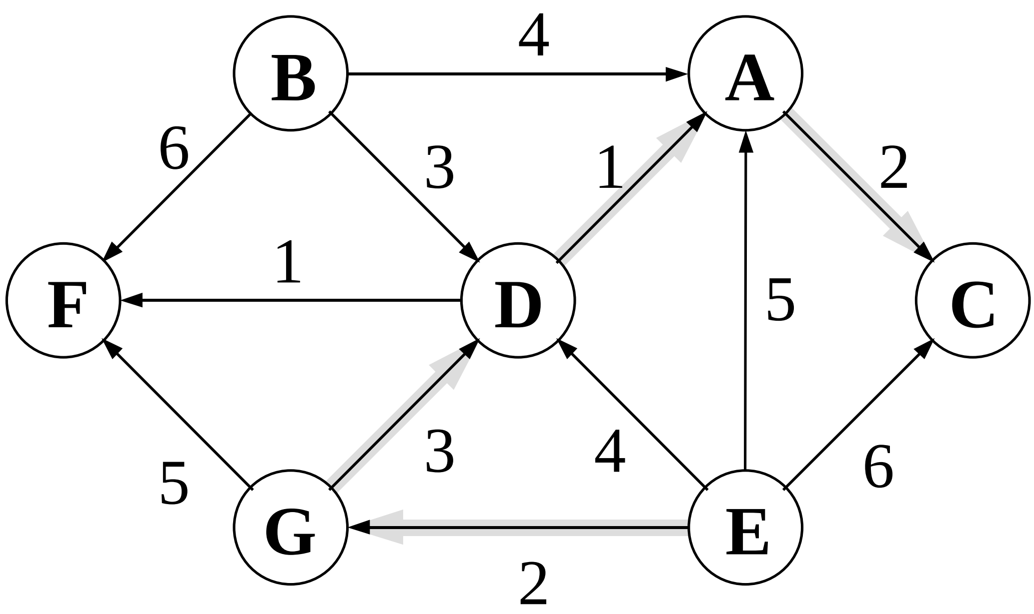
	i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7
A	1			1	3	2	5	
B	2					5		7
C	3	1					8	
D	4						4	1
E	5	2	5		5			
F	6	5			4			
G	7		7		1	3	5	

	i \ j	A 1	B 2	C 3	D 4	E 5	F 6	G 7	OG	OC
A	1	2	7	1	3	2	5	4	**(22)**	(7)
B	2	7	10	8	8	5	**12**	7	47	12
C	3	1	8	2	4	3	6	5	27	8
D	4	6	8	7	2	4	4	1	30	8
E	5	2	5	3	5	4	7	6	28	(7)
F	6	5	**12**	6	4	7	8	5	39	12
G	7	5	7	6	1	3	5	2	27	(7)
DG		26	47	31	25	**(24)**	39	28		
DC		(7)	12	8	8	**(7)**	12	(7)		

Direct- and Overall Access Tables with <u>G</u>ravity-points and <u>C</u>enters as „<u>O</u>rigins” and/or „<u>D</u>estinations” together with Span

THE LONGEST SPAN

$$\left(M = -\infty;\ \varphi\left(a_{ij}^{k-1}, a_{ik}^{k-1}, a_{kj}^{k-1}\right) = \max\left\{ a_{ij}^{k-1},\ a_{ik}^{k-1} + a_{kj}^{k-1} \right\} \right)$$

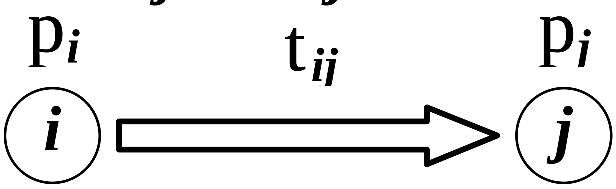


		A	B	C	D	E	F	G
	i \ *j*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
A	*1*			2^{2}				
B	*2*	4^{4}		6	3^{3}		6^{6}	
C	*3*							
D	*4*	1^{1}		3			1^{1}	
E	*5*	6^{5}		$\mathbf{8}^{6}$	5^{4}		7	2^{2}
F	*6*							
G	*7*	4		6	3^{3}		5^{5}	

GTM (General Time Model)

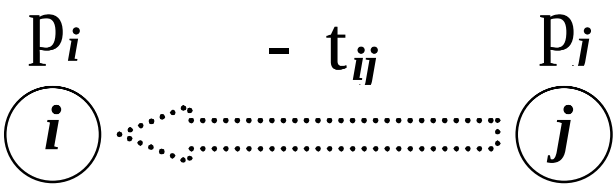
Homogenizing relations

$$t_{ij} \text{ £ } p_j - p_i$$



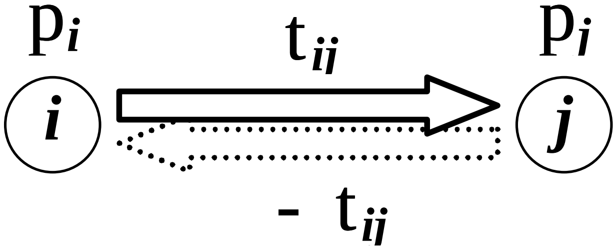
$$p_j - p_i \text{ £ } t_{ij} \quad /\cdot(-1)$$

$$-\tau_{ij} \leq \pi_i - \pi_j$$



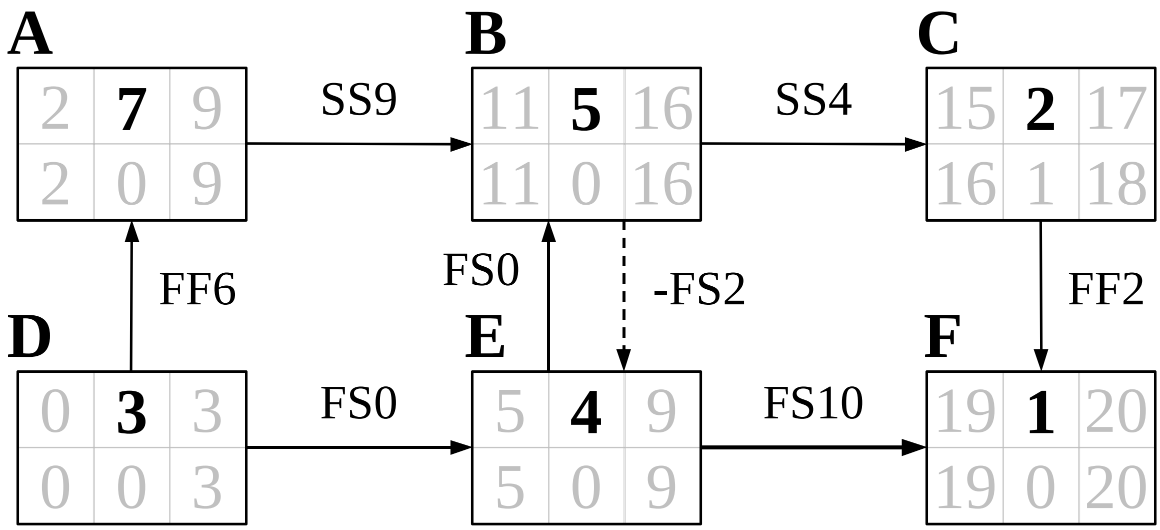
$$p_j - p_i = t_{ij}$$

$$\tau_{ij} \leq \pi_j - \pi_i \leq \tau_{ij}$$

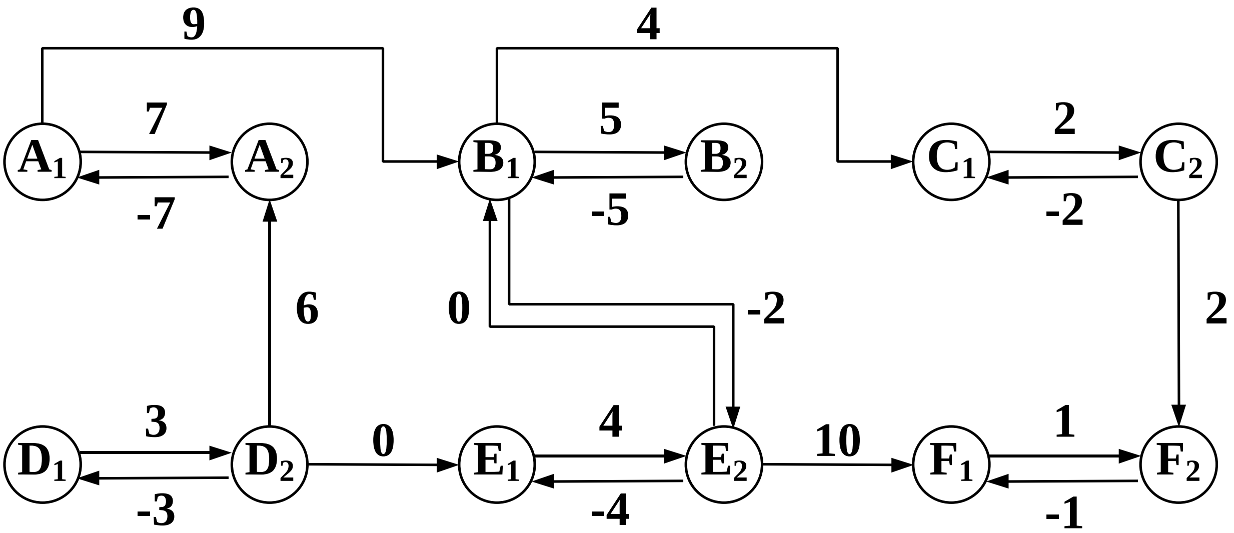


MPM/PDM → GTM

MPM/PDM network model
with heterogeneous boundaries



Transforming MPM/PDM to GTM
with homogeneous boundaries



Modified/new terms

Positive Source :

A nod being origin of at least one directed edge with non-negative weight but not terminal point of any directed edges with non-negative weight

Positive Sink :

A nod being terminal point of at least one directed edge with non-negative weight but not origin of any directed edges with non-negative weight

Positive/Negative/Zero loop :

According to the sum of weights along the loop

Recognition :

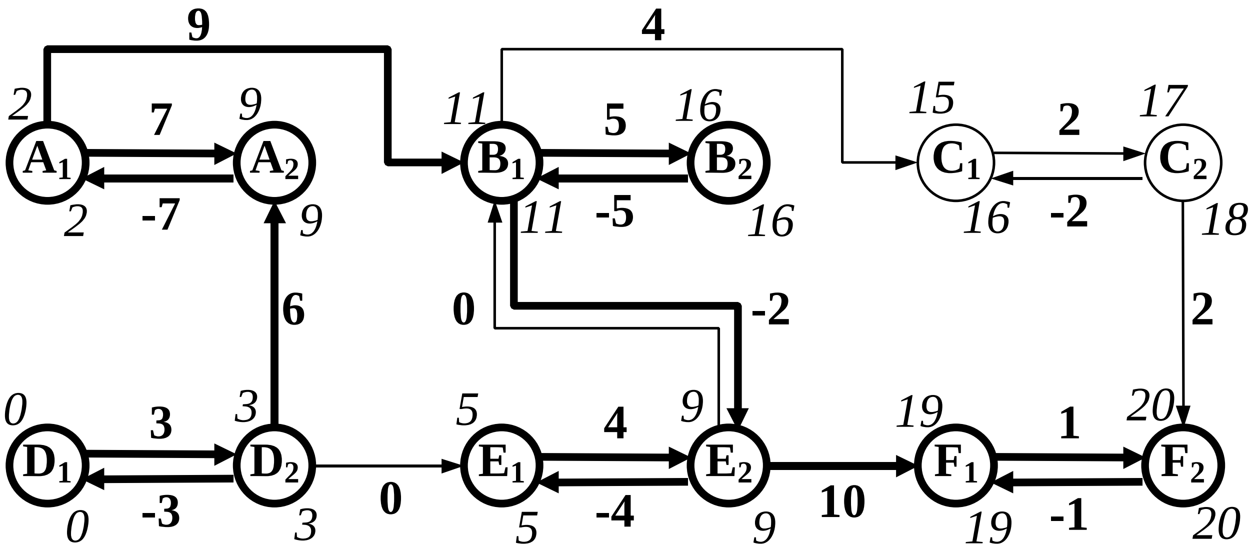
Weights of first and of last edge of the Longest Path are surely non-negative !

Critical Path :

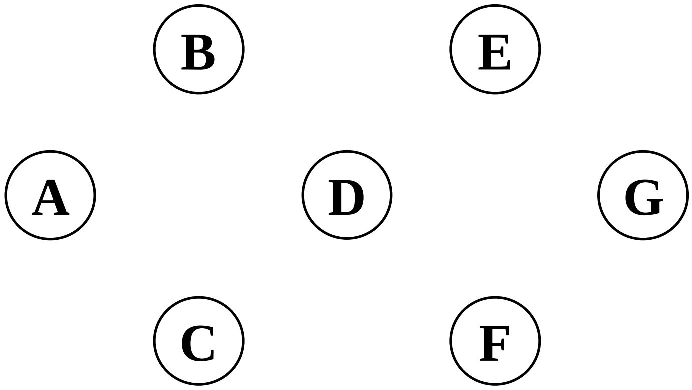
<u>Sub-graph</u> constituted by the longest paths between positive source<u>s</u> and positive sink<u>s</u>

Graph restrictions : - *(Positive loop !)*

Assigning Time Potentials



p^f \ p^n		20		20								20	
		A_1	**A_2**	**B_1**	**B_2**	**C_1**	**C_2**	**D_1**	**D_2**	**E_1**	**E_2**	**F_1**	**F_2**	p^{max}
0	**A_1**	0	7	9	14	13	15			3	7	17	18	2
	A_2	-7	0	2	7	6	8			-4	0	10	11	~~20~~ 9
	B_1			0	5	4	6			-6	-2	8	9	11
	B_2			-5	0	-1	1			-11	-7	3	4	~~20~~ 16
	C_1					0	2					3	4	16
	C_2					-2	0					1	2	18
0	**D_1**	2	9	11	16	15	17	0	3	5	9	19	**20**	0
	D_2	-1	6	8	13	12	14	-3	0	~~0~~2	6	16	17	3
	E_1			4	9	8	10			0	4	14	15	5
	E_2			0	5	4	6			-4	0	10	11	9
	F_1											0	1	19
	F_2											-1	0	20
	p^{min}	~~0~~ 2	9	11	16	15	17	0	3	5	9	19	20	



		A	B	C	D	E	F	G
	i \ j	1	2	3	4	5	6	7
A	1							
B	2							
C	3							
D	4							
E	5							
F	6							
G	7							

i \ j	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

i \ j	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

i \ j	1	2	3	4	5	6	7
1							
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3							
4							
5							
6							
7							

i \ j	1	2	3	4	5	6	7
1							
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4							
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6							
7							

i \ j	1	2	3	4	5	6	7
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i \ j	1	2	3	4	5	6	7
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i \ j	1	2	3	4	5	6	7
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i \ j	1	2	3	4	5	6	7
1							
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7							